Stat 444
Advanced Long-term Actuarial Math

Lecture 2: Long Term Health Insurance Coverages

Disability Income Insurance (DI)

This coverage is intended to replace income in the event that the insured cannot work due to a disability.

It is typically a level premium product, but premiums are waived upon disability.

This insurance is sometimes offered as a group policy through employers.

Benefits are often based on the insured's salary.

- Though the benefit is typically set at less than 100% of salary, in order to encourage the insured to return to work when able.
- There may be lesser benefit levels paid for a partial disability.
- There may be inflation adjustments in the payments.

The term of the payments may be 2 years, 5 years, or to age 65.

Some Important Terms

A **waiting period** or **elimination period** (of e.g., 30 days, 60 days, 180 days) is usually included; this is the length of time between the onset of disability and the start of payments.

If a second disability claim occurs shortly after the end of a first claim, the two claims may be combined, i.e., the second claim may be considered to be an extension of the first claim rather than a new one.

- In this case, a second elimination period would not be required.
- The **off period** specifies how much time must pass between claims in order for a new claim to be considered distinct.

A disability may be based on the ability to do one's current job ("own occ"); a much stricter definition would require the insured not to be able to do any job ("any occ") to be considered disabled.

Long Term Care Insurance (LTC)

Benefits for LTC insurance are triggered when the insured cannot perform (usually) 2 or 3 of the following 6 **ADLs (Activities of Daily Living)**:

- Bathing
- Dressing
- Eating
- Toileting
- Continence
- Transferring

The term of benefits may be fixed (e.g., 2 or 5 years) or indefinite.

Benefits may follow a "reimbursement" model where the insurance pays the caregiver directly, or the benefit may be a fixed amount per month / year.

- Inflation adjustments are sometimes included in benefit amounts.

It's common for life insurance policies (especially perm) to include some kind of LTC coverage, or accelerated benefits for LTC.

Critical Illness Insurance (CI)

Critical Illness (CI) insurance pays a lump sum upon the diagnosis of a disease from a specified list.

- The benefit is one lump sum, after which the policy expires.
- This type of insurance may be included as a rider on a life insurance policy, in the form of accelerated benefits.

This is typically a level premium product, and may offer a (partial) return of premium feature.

Continuing Care Retirement Communities (CCRC)

A **Continuing Care Retirement Community (CCRC)** is a living facility designed for seniors, and usually having different levels of care available:

- **Independent Living Units (ILUs)** are similar to apartments with little or no external care provided (but convenient access to care if needed)

- **Assisted Living Units (ALUs)** provide some (mostly non-medical) support for residents needing help with one or more ADLs.

- A **Skilled Nursing Facility (SNF)** offers ongoing medical care for residents needing more substantial care.

- **Memory Care Units (MCUs)** are often separated from other units, and allow for more specialized care for patients with dementia and other mental impairments.

Funding Models for CCRCs

Some of the more popular funding models include:

- **Full life care (Type A)** arrangements require a large upfront fee, with a level monthly premium thereafter. All residential needs, health care costs, and personal assistance are covered by these costs without additional fees.

- **Modified life care (Type B)** contracts pay lower upfront and monthly fees, but residents are charged additional fees for various services. (Though these additional fees would typically be less than full market cost.)

- **Fee-for-service (Type C)** have much lower entry fees and monthly payments, but residents pay market rates for any health care they receive.

There are many variations on these themes, including partial ownership models and joint membership models for couples.

A **continuous sojourn annuity** is an annuity paying continuously for as long as an individual remains (continuously) in their current state. The annuity ceases payment when the individual leaves their current state, even if they subsequently return to it.

For example, a continuous $n-$year term sojourn annuity issued to (x) in state i, payable for the time the annuitant stays continuously in state i (but for a maximum of n years), is given by

$$\bar{a}^{\overline{ii}}_{x:\overline{n}|} = \int_0^n {}_tp^{\overline{ii}}_x \, e^{-\delta t} \, dt$$

DI Example

Consider a 5-year DI product issued to (x), who is healthy at issue, that pays at a continuous rate of 1000 per year while the insured is disabled.

$$1000 \int_0^5 {}_tp_x^{01} e^{-\delta t} dt = 1000 \int_0^5 e^{-\delta t} {}_tp_x^{00} \mu_{x+t}^{01} \bar{a}_{x+t}^{\overline{11}} \, dt \quad \text{(forever)}$$

(a) Give an expression for the EPV of benefits for this DI product.

Now repeat the example assuming that:

(b) The product has a maximum benefit payment period of 2 years per disability.
$$1000 \int_0^5 e^{-\delta t} {}_tp_x^{00} \mu_{x+t}^{01} \bar{a}_{x+t:\overline{2}|}^{\overline{11}} dt$$

(c) The product has a half-year waiting period prior to commencement of payments.
$$1000 \int_0^5 e^{-\delta t} {}_tp_x^{00} \mu_{x+t}^{01} \left[\bar{a}_{x+t}^{\overline{11}} - \bar{a}_{x+t:\overline{0.5}|}^{\overline{11}} \right]$$

$$e^{-0.5\delta} \, {}_{0.5}p_{x+t}^{\overline{11}} \, \bar{a}_{x+t+0.5}^{\overline{11}}$$

(d) The product has both the maximum benefit payment and the waiting period.
$$1000 \int_0^5 e^{-\delta t} {}_tp_x^{00} \mu_{x+t}^{01} \left[\bar{a}_{x+t:\overline{2}|}^{\overline{11}} - \bar{a}_{x+t:\overline{0.5}|}^{\overline{11}} \right]$$

A Model for DI
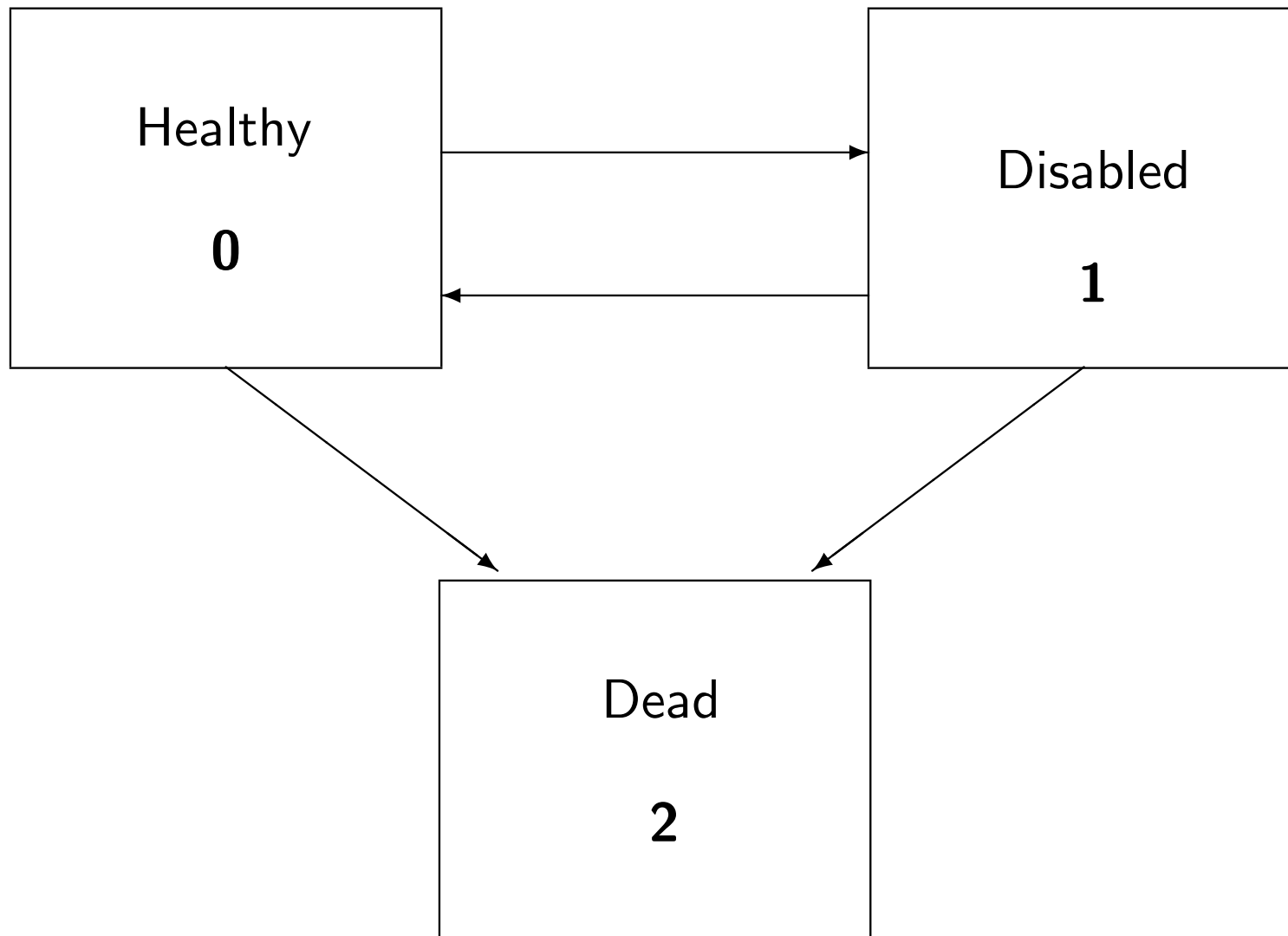
LTC Example

Consider a LTC product issued to (x), having lifetime benefits, where the benefits are payable continuously and vary by level of impairment (Impaired, Severely Impaired, Cognitive Impairment). There are no death benefits, and premiums are payable continuously while the insured is Active.

- Give expressions for the Kolmogorov Forward Equations, assuming the insured is currently in state 1, with boundary conditions.

$$_{t+h}p_x^{ij} \approx {}_tp_x^{ij} + h \sum_{k=0,k\neq j}^{n} \left({}_tp_x^{ik}\mu_{x+t}^{kj} - {}_tp_x^{ij}\mu_{x+t}^{jk} \right)$$

$$_{0.25}p_x^{12} \approx {}_0p_x^{12} + h\sum(\cdots\cdots)$$

$$= 0 + 0.25 \left(\mu_x^{12} \right)$$
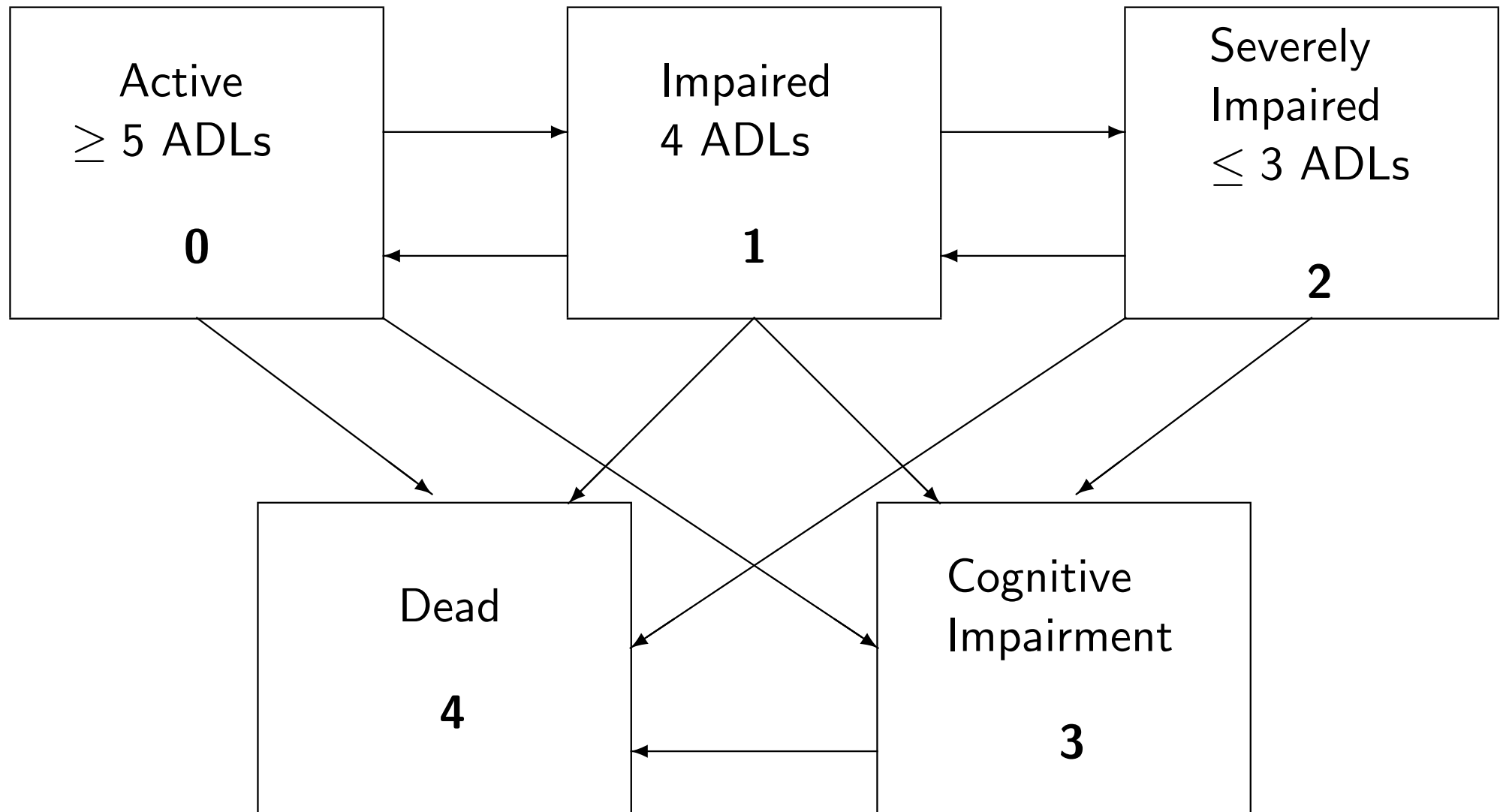
- Give an expression for Thiele's Differential Equation for $\frac{d}{dt}{}_tV^{(1)}$, ignoring expenses.

$$_{0.25}p_x^{11} \approx 1 + 0.25 \left(-\mu_x^{10} - \mu_x^{12} - \mu_x^{13} - \mu_x^{14} \right)$$

$$\frac{d}{dt}{}_tV^{(i)} = \delta_t \, {}_tV^{(i)} - B_t^{(i)} - \sum_{j=0,j\neq i}^{n} \mu_{x+t}^{ij} \left(S_t^{(ij)} + {}_tV^{(j)} - {}_tV^{(i)} \right)$$

$$\frac{d}{dt}{}_tV^{(1)} = \delta_t \, {}_tV^{(1)} - B_t^{(1)} - \sum \mu^{1j} \left({}_tV^{(j)} - {}_tV^{(1)} \right)$$

A Model for LTC

CI Example

Consider a fully continuous whole life insurance policy issued to (40) with a partially accelerated benefit for critical illness. Benefits consist of:

- A death benefit of 100,000 is payable upon death.
- In the event of a critical illness diagnosis, a 50,000 lump sum is paid, and the death benefit is subsequently reduced by this amount.

Premiums are payable continuously while the insured is healthy. Expenses are $500 at issue and 3% of gross premiums.

1. Give an expression for the gross premium rate using the equivalence principle.
2. Give an expression for the gross premium reserve at time 10, if the insured is healthy.
3. Give an expression for the gross premium reserve at time 10, if the insured is critically ill.
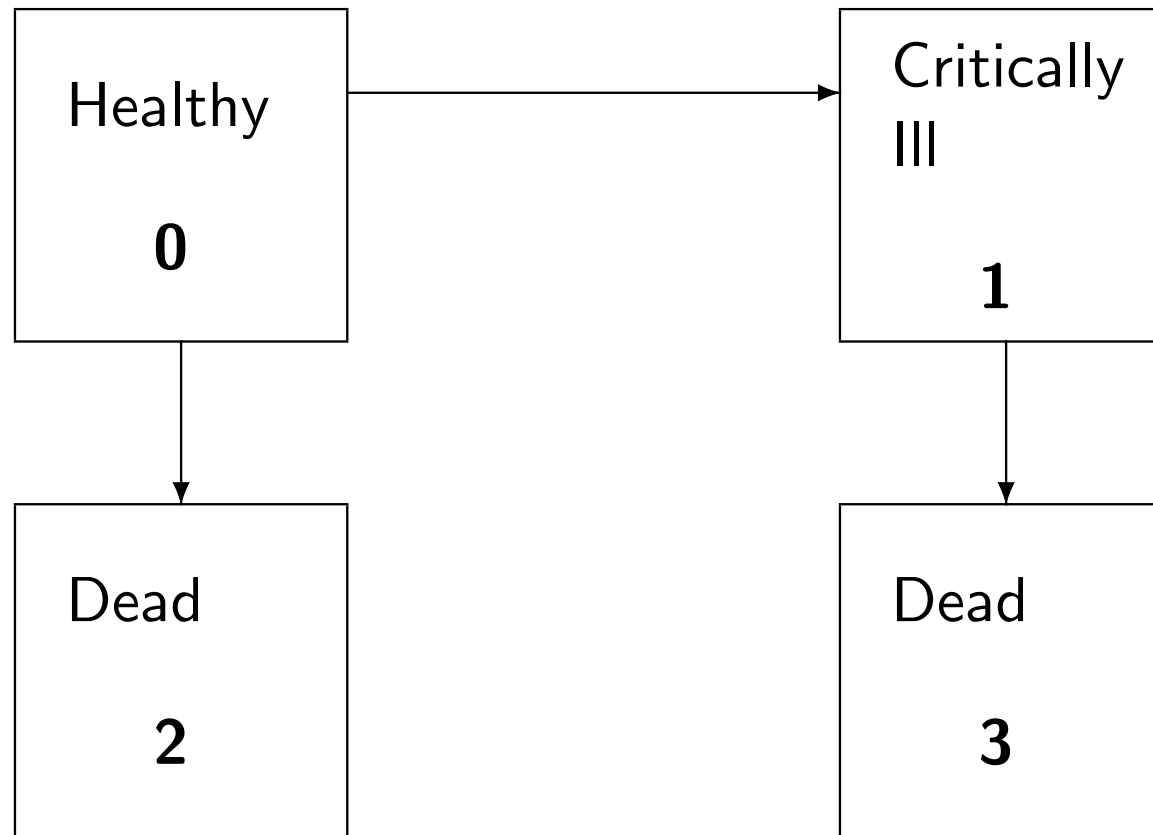
The model above could be used for a policy with a death benefit that's accelerated in the event of a critical illness diagnosis; this necessitates the two different Dead states.

CCRC Example

Consider a CCRC providing care at three levels: ILU, ALU, and SNF. The CCRC is a full life care (Type A) arrangement, requiring an entry fee and a level monthly fee regardless of the current care level being given. (Though the actual monthly cost of the benefits will vary by level of care.) The only death benefit is a refund of the entry fee at the end of the month of death.

Write monthly recursion equations for $_tV^{(j)}$ for $j = 0, 1, 2$, assuming monthly fees, expenses, and care benefits are all paid at the start of each month.

A Model for CCRC